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EFFECT OF TOOTH PROFILE MODIFICATION ON LUBRICATION PERFORMANCE OF SPUR GEAR PAIRS

ABSTRACT

Tooth profile modification is a familiar method to improve load capacity and avoid tooth failures, however, the lubrication performance of a meshing gear pair surfaces would be affected due to the change of gear micro-geometry. Firstly, a simplified model for a meshing gear pair is established. Tooth load, rolling velocity and contact geometry vary as the gear teeth come into action. Later, a transient elastohydrodynamic lubrication (EHL) model of an engaged spur gear pair is developed. Film thickness and pressure could be obtained by utilizing multigrid method. Then, influence of tooth profile modification (TPM) on the lubrication performance are investigated based on the proposed EHL model. At last, the lubrication performance under different operating conditions are studied.

AUTHORS AND INSTITUTIONS

Junbin Lai[1], Yanfang Liu[1*], Xiangyang Xu[1]

[1]Department of Automotive Engineering, School of Transportation Science and Engineering, Beihang University, Beijing 100191, PR China

[*]Corresponding Author

INTRODUCTION

Due to the advantages of high transmission efficiency, large load capacity and good reliability, gear trains are widely used in variety mechanical systems. Among all kinds of gear train, the involute spur gear is the simplest. Since the meshing involute spur gear is typically non-stationary, the analysis of elastohydrodynamic lubrication (EHL) behavior is very complicated. It is the case that not only the load, velocity also curvature of meshing point vary as the gear teeth come into action. Moreover, during the meshing process, the rolling velocity and sliding velocity also vary along the line of action [1]. As a result, it is difficult to obtain the transient EHL behavior of involute spur gear. Fortunately, Yang and Wen [2] have derived the generalized Reynolds equation which can be applied to line contact, point contact, Newtonian fluid, non-Newtonian fluid, transient analysis and steady analysis. Furthermore, Lubrecht et,al. [3] solved the EHL problem by using multigrid method, which is confirmed to be efficient and stable.

Compared with helical gears, even though the spur gears have better load capacity, the noise and vibration are more intensive due to the sudden change of gear tooth at the region between the alternating numbers of tooth pairs in mesh. Not only the load capacity and reliability but also the fatigue life would be affected by the noise and vibration. To work out the issue, tooth profile modification (TPM), removing materials from tooth surface, is an effective method. The amount of TPM is always micrometer order, which has infinitesimally small impact on the geometry of gear tooth. But the gear tooth load would be changed greatly, and proper modification would smooth the gear tooth load well. Variation gear tooth load has influence on the dynamic lubrication process, so that it is essential to investigate the effect of tooth modification on the lubrication performance. Furthermore, the working condition for gear pairs might not be constant in practice, the effect of operating speed and applied torque is studied.

MODELING

Model of gear tooth load

Tooth tip relief (TTR) is among the most common TPM method, and will be adopted in this paper. Fig.1 shows the schematic of TTR applied on gears. The formulation of tip relief shape can be expressed as:

$$\gamma_a = \beta_e - \beta_s \tag{1}$$

$$C = C_a\left(\beta - \beta_s\right)/\gamma_a \tag{2}$$

where β stands for angular displacement, subscript e and s respectively for ending and starting point for tip relief, C_a stands for amount of TTR.

To express modification angle, the highest point of single tooth contact is selected as reference. We define the normalized modification angle as:

$$A_n = \gamma_a/\gamma_t \tag{3}$$

where A_n represents normalized modification angle, γ_t donates reference modification angle, and can be expressed as $\gamma_t=2\pi(\varepsilon-1)/z$, ε, z refers to the contact ratio and the number of gear teeth, respectively.

For the purpose of improving dynamic tooth load, TTR could be considered as an intended tooth profile error. Recently, Chen and Shao [4] investigate the relationship between gear profile errors and the load sharing factor of gear tooth pair, which can be written as:

$$Lsf_i = K_i\left(1+\sum_{j=1}^{N}K_j\tilde{E}_{ij}\right)\Bigg/\left(F\sum_{j=1}^{N}K_j\right) \tag{4}$$

Detailed meanings of different symbols in the equation can be found in Ref. [3].

EHL model of spur gear

Fig.2 shows the diagram of a meshing gear pair. Where N_1N_2 is the line of action; point K is the transient contact point; point P is the pitch point; the length between point K and P is $KP=s$. R_{b1}, R_{b2} is base circle radius of pinion and gear, respectively.

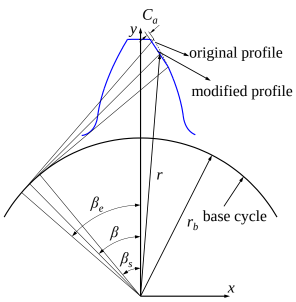

Fig. 1. The schematic of tip relief.

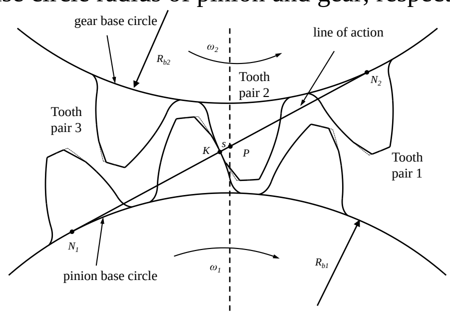

Fig. 2. diagram of a meshing gear pair

Based on the work of Yang and Wen [2], the Reynolds equation for transient line problems of Newtonian fluid can be written as:

$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\eta}\frac{\partial p}{\partial x}\right)=12u_r(t)\frac{\partial(\rho h)}{\partial x}+12\frac{\partial(\rho h)}{\partial t} \tag{5}$$

where p is film pressure; h is film thickness; η is lubricant viscosity; ρ is lubricant density; x is coordinate variable; t is time variable.

RESULTS AND DISCUSSIONS

In order to reduce the total number of parameters needed to be solved the problem, dimensionless parameters are introduced:

$$\begin{aligned} X &= x/b \qquad H = hR_0/b^2 \\ P &= p/p_H \quad C_w(t) = w(t)/w_0 \end{aligned} \tag{6}$$

where b is the half-width of the Hertzian contact at pitch point, $\sqrt{8R_0w_0/(\pi E')}$; R_0 is the synthetic curvature radius at pitch point, p_H is the maximum Hertzian pressure at pitch point, $\sqrt{w_0E'/(2\pi R_0)}$; w_0 is the gear tooth laod at pitch point.

A uniform grid by the finite difference method is adopted to solve Reynolds equation. The mutigrid method utilizing the Gauss-Seidel relaxation and the Jacobi dipole relaxation is employed to improve the convergence rate. Six levels of grids are used with 961 nodes on the finest level. The computation domain are $X_{in}=-4.6$ and $X_{out}=1.4$. Amount of TTR and normalized modification angle are assumed to be identical for pinion and gear. Effect of TTR on the curvature radius is neglected. The gear parameters and lubricant properties used in this paper are exhibited in Table 1.

Table 1 gear parameters and lubricant properties

Number of teeth, z_1 and z_2, (pinion:gear)	22:32
Module, m(mm)	2
Nominal pressure angle (deg)	25
Teeth width (mm)	20
Comprehensive elastic modulus, E', (GPa)	226
Viscosity of lubricant, η_0, (Pa s)	0.08

Effects of TTR

Two cases with C_a=10μm, A_n=1 and C_a =10μm, A_n=0.3 are analyzed. The operating speed of the pinion is 2000rpm and applied torque is 100Nm. Fig.3 (a) shows the time-varying dimensionless load coefficient defined in Eq. (6). Fig.3 (b) shows the time-varying central film pressure. Fig.3 (c) and Fig.3 (d) show the time-varying central and minimum film thickness. Sudden change of gear tooth load occurs between region A and region B without modification, but TTR can smooth the gear tooth load as illustrated in Fig.3 (a). It can be found that central pressure increases firstly, then decreases and increases again at gear tooth load sudden increases region without modification. An equilibrium state is unable to get until gear tooth load sudden decrease. This phenomenon can also be found in central film thickness and minimum film thickness. When time-varying gear tooth load get improved after modification, sudden change of gear tooth load is eliminated, which makes central pressure varies slightly at alternating meshing tooth pairs region. Furthermore, better time-varying gear tooth load gets more steady film thickness at the region. After modification, the absolute slope of gear tooth load varies with *s* becomes more rapidly at region A and region C. With the increasing of the slope, central pressure first increase, then decreases and keeps increasing at region A, and vice versa at region C. The central film thickness has the same phenomenon. The minimum film thickness would decrease on the whole trends as the increasing of the slope at region A. While at region C, minimum film thickness keeps raising as the absolute slope increasing.

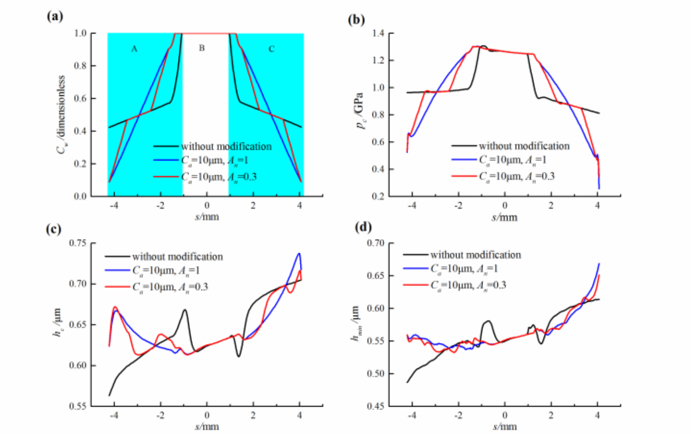

Fig. 3. Effects of tooth profile modification on: (a) gear tooth load, (b) central film pressure, (c) central film thickness and (d) minimum film thickness.

Effects of operating speed and applied torque

Four different operating speeds, 1000rpm, 2000rpm, 3000rpm and 4000rpm, are analyzed, and the applied torque is identical, 100Nm. The gear tooth remain unmodified. Fig.4 (a-c) shows the dimensionless central pressure, central film thickness and minimum film thickness under different operating speed. The fluctuation magnitude of central pressure get higher at gear tooth load sudden change region as the operating speed increases, but the effect is quite slightly as illustrated in Fig.4 (a). Fig.4 (b-c) indicates that dimensionless central film thickness and minimum film thickness increase at any mesh position as the operating speed increases. Furthermore, similar with central pressure, magnitude of film thickness fluctuation get higher at gear tooth load sudden change region.

Subsequently, three different torques applied at pinion are analyzed, 100Nm, 150Nm, 200Nm. The operating speed keeps 2000rpm and the gear tooth remains unmodified. Fig.5 (a-c) shows the dimensionless central pressure, central film thickness and minimum film thickness under different applied torques. Fig.5 (a) shows that dimensionless central pressure is approximate for different applied torques. The magnitude of central pressure fluctuation at gear tooth load sudden change region is higher for smaller applied torque. Fig.5 (b-c) shows that dimensionless central film thickness and minimum film thickness get thinner as applied torque increases at any mesh position. Furthermore, smaller applied torque obtains higher dimensionless film thickness fluctuation magnitude at gear tooth sudden change region as displayed in Fig.5 (b-c).

Fig.6 shows the dimensionless central pressure, central film thickness and minimum film thickness at three characteristic meshing positions under different working conditions, where operating speed varies from 50rpm to 4000rpm. It can be found that higher applied torque achieves higher dimensionless central pressure and thinner film thickness. But the effects of applied torque and operating are slightly. Furthermore, dimensionless central pressure varies as operating speed nonlinearly, however, dimensionless central film thickness and minimum film thickness vary linearly as operating speed when it exceeds 500rpm. Meanwhile, dimensionless minimum film thickness varies more slowly than dimensionless central film thickness.

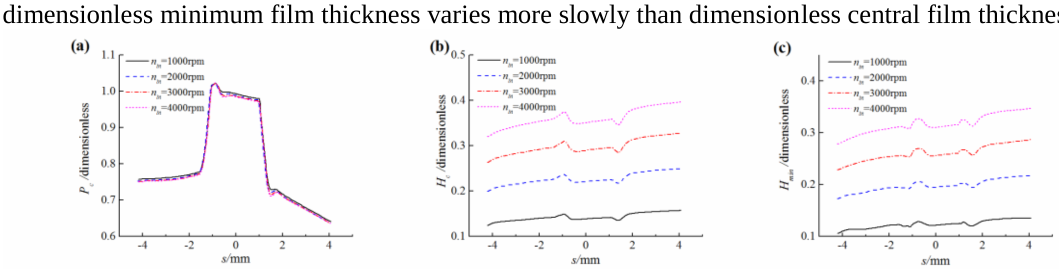

Fig. 4. Effects of operating speed on (a) dimensionless central pressure, (b) dimensionless central film thickness and (c) dimensionless minimum film thickness.

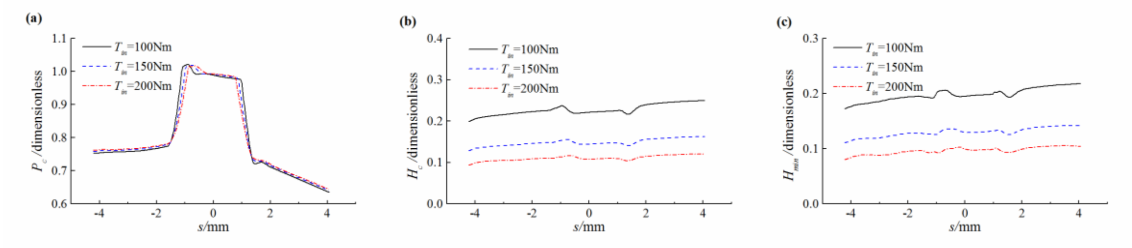

Fig.5. Effects of applied torque on (a) dimensionless central pressure, (b) dimensionless central film thickness and (c) dimensionless minimum film thickness.

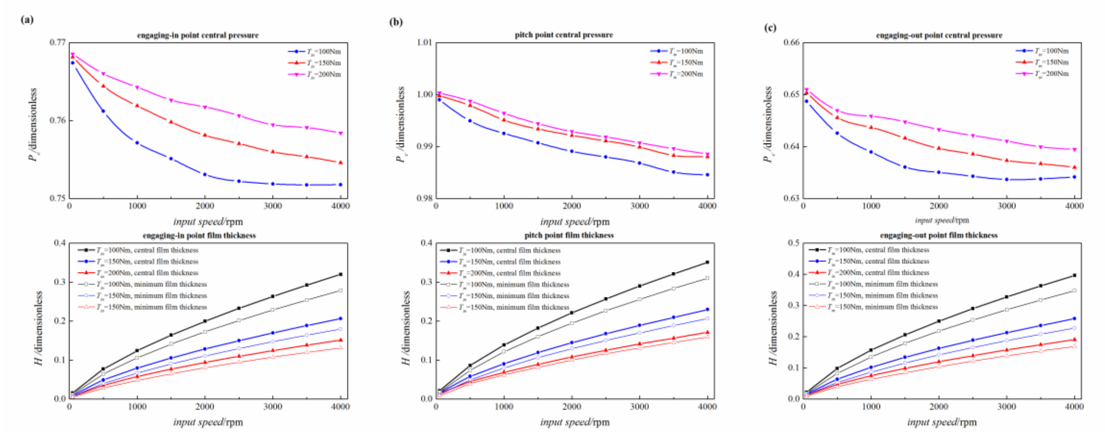

Fig. 6. Dimensionless central pressure and film thickness at (a) engaging-in point, (b) pitch point and (c) engaging-out point under different applied torques and operating speeds.

Conclusion

1. Sudden change of gear tooth load would cause fluctuation of central film pressure, central film thickness and minimum film thickness. The faster gear tooth load changes, the more significant fluctuation appears.
2. Proper TTR can improve the time-varying gear tooth load. Smooth gear tooth load can make central pressure varies smooth, and minimum film thickness decrease on the whole trends in region A. The fluctuation of central film pressure, central film thickness and minimum film thickness could be reduced in a certain degree.
3. Variation of operating speed and applied torque has slightly influence on the dimensionless central film pressure. The larger applied torque would obtain greater central film pressure and thinner film thickness at three character point. The larger operating speed would get thicker dimensionless film thickness. Dimensionless film thickness varies linearly as operating speed, while dimensionless film pressure varies non-linearly.

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KEYWORDS

Elastohydrodynamic lubrication; Spur gear; Tooth profile modification